Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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1 Which of the following is equal to $\dfrac{1}{\sqrt{5}-1}$?

(A) $\sqrt{5}-1$

(B) $\dfrac{\sqrt{5}+1}{4}$

(C) $\dfrac{\sqrt{5}-1}{4}$

(D) $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$

2 What is the solution(s) to the equation $|2k+1| = k+1$?

(A) 0 only

(B) 0 or $\dfrac{2}{3}$

(C) 0 or $\dfrac{-2}{3}$

(D) 0 or $\dfrac{-1}{2}$

3 A parabola is concave down and its vertex is $(2,\ 0)$.
Which statement about the discriminant (Δ) of the parabola is correct?

(A) $\Delta > 0$

(B) $\Delta = 0$

(C) $\Delta < 0$

(D) $\Delta \leq 0$

4 If $f'(x) = 2\cos(5x)$ and c is a real constant, then what is $f(x)$ equal to?

(A) $-\frac{2}{5}\sin(5x) + c$

(B) $\frac{2}{5}\sin(5x) + c$

(C) $-10\sin(5x) + c$

(D) $10\sin(5x) + c$

5 What are the coordinates of the focus of the parabola $(x+3)^2 = -12y$?

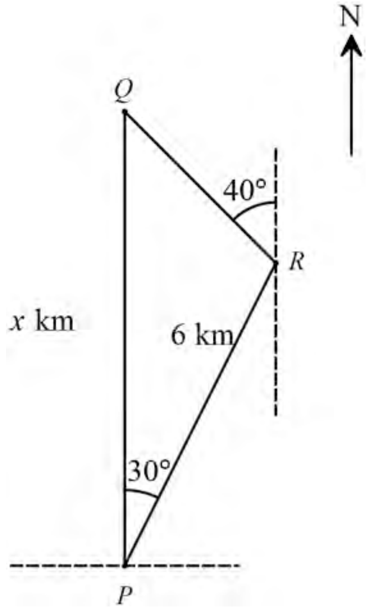
(A) $(-3, -3)$

(B) $(-3,\ 3)$

(C) $(0,\ -3)$

(D) $(0,\ 3)$

6 A ship leaves a port, P, and sails 6 km on a heading of $\text{N}30^\circ\text{E}$ to position R. It then heads $\text{N}40^\circ\text{W}$ until it reaches a port, Q, which is directly north of P.



Which equation represents the distance x km from P to Q?

(A) $\dfrac{x}{\sin 40^\circ} = \dfrac{6}{\sin 70^\circ}$

(B) $\dfrac{x}{\sin 30^\circ} = \dfrac{6}{\sin 40^\circ}$

(C) $\dfrac{x}{\sin 40^\circ} = \dfrac{6}{\sin 30^\circ}$

(D) $\dfrac{x}{\sin 110^\circ} = \dfrac{6}{\sin 40^\circ}$

7 If $y = 2\tan(2x)$, then which expression represents $\dfrac{dy}{dx}$?

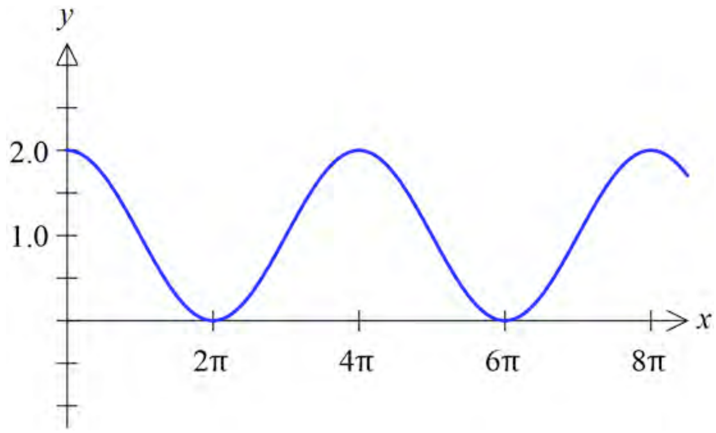
(A) $\dfrac{1}{\cos^2(2x)}$

(B) $\dfrac{2}{\cos^2(2x)}$

(C) $\dfrac{4}{\cos^2(x)}$

(D) $\dfrac{4}{\cos^2(2x)}$

8 The diagram below shows part of the graph of a circular function.



Which equation represents the graph shown?

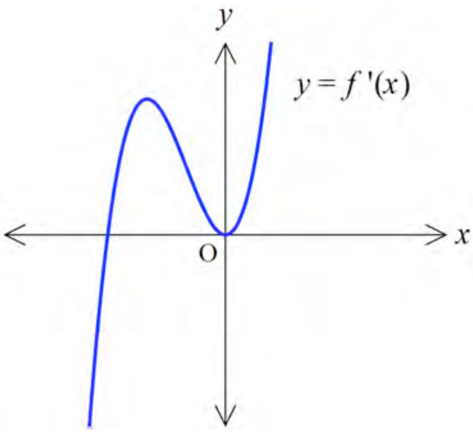
(A) $y = 1 + \sin(x)$

(B) $y = 1 + \sin\left(\dfrac{x}{2}\right)$

(C) $y = 1 + \cos(x)$

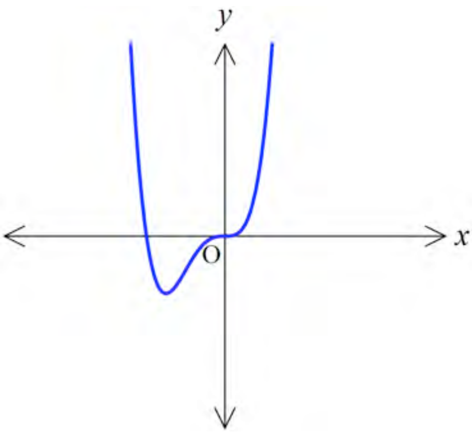
(D) $y = 1 + \cos\left(\dfrac{x}{2}\right)$

9 The graph of a curve $y = f'(x)$, is shown below.

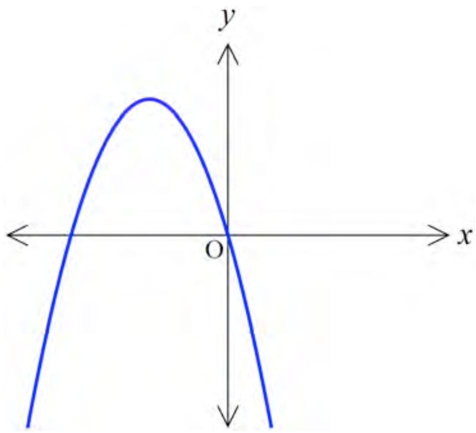


Which one of the following is most likely to be the graph of the function of $f(x)$?

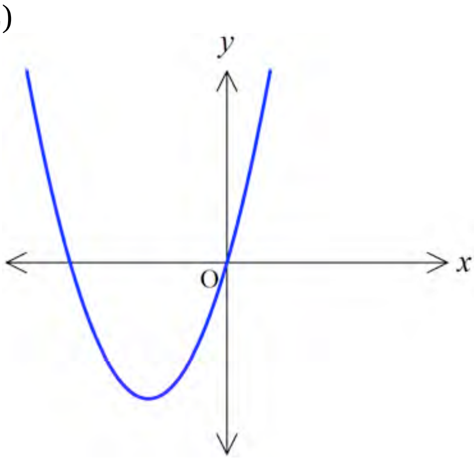
(A)



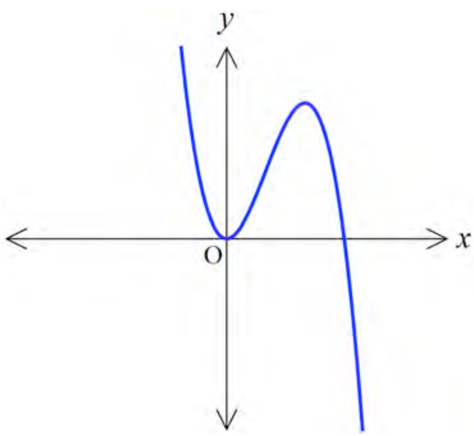
(B)



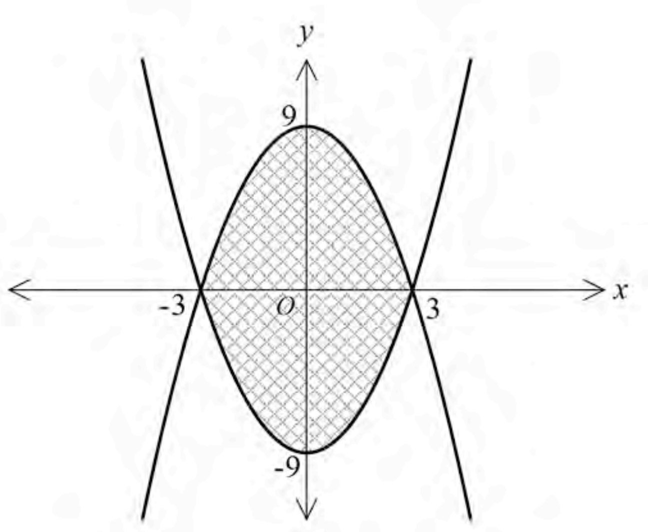
(C)



(D)



10 The area of the region enclosed between the equations $y = x^2 - 9$ and $y = 9 - x^2$ is shaded in the diagram. Which integral could be used to calculate the shaded area?



(A) $\int_{-3}^{3} 2x^2 - 18\,dx$

(B) $2\int_{0}^{3} 18 - 2x^2\,dx$

(C) $\int_{-9}^{9} 2x^2 - 18\,dx$

(D) $\int_{-9}^{9} 18 - 2x^2\,dx$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Solve $x^2 - 3 = 3x + 1$. **2**

(b) Solve the simultaneous equations **2**

$$\begin{aligned} 3x - y &= -7 \\ 5x + 2y &= 3 \end{aligned}$$

(c) Differentiate $\dfrac{2x^3}{4x+2}$. **2**

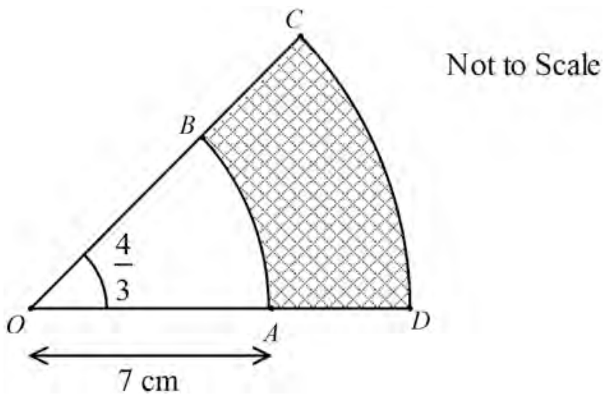
(d) Evaluate $\displaystyle\int_0^{\pi} \sin 2x \ dx$. **2**

(e) Evaluate $\displaystyle\int_0^1 \frac{5}{\sqrt{e^x}} dx$. **3**

Question 11 continues on page 8

(f) The angle of a sector in a circle of radius 7 cm is $\frac{4}{3}$ radians, as shown in the diagram.

The points A and B lie on OD and OC respectively and AB is an arc of a circle.
O is the centre of the circle.
The area of the shaded region $ABCD$ is 48 cm^2.



(i) Find the distance OD. **2**

(ii) Find the perimeter of the shaded region. **2**

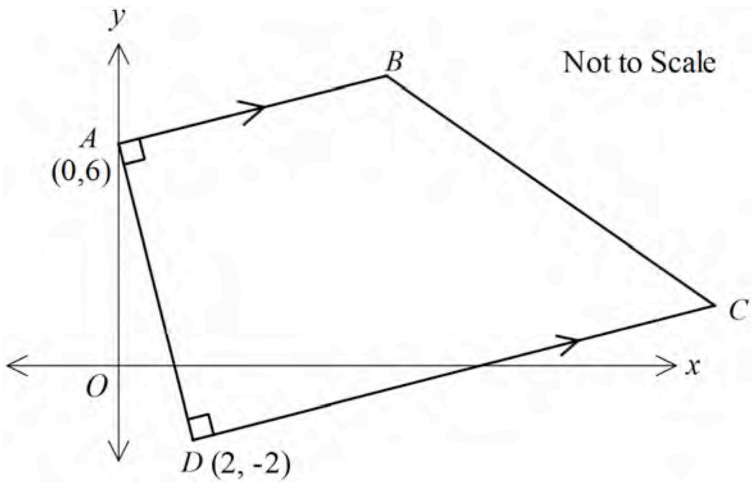
End of Question 11

Question 12 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Calculate the limiting sum of the infinite geometric series given by **2**

$$2 - 1 + \frac{1}{2} \ldots$$

(b)



The diagram shows a trapezium *ABCD* in which *AB* is parallel to *DC* and *BA* is perpendicular to *AD*. The length of *DC* is twice the length of *AB*.
The point *A* is $(0, 6)$ and the point *D* is $(2, -2)$.

(i) Show that equation of *AB* is $x - 4y + 24 = 0$. **2**

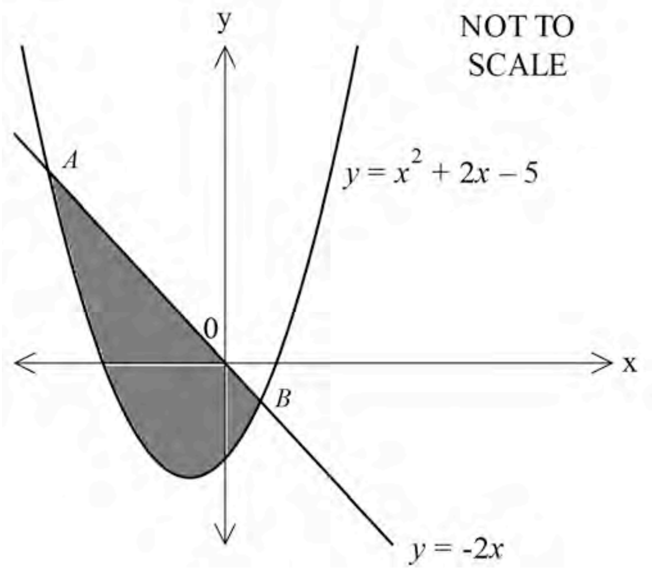
(ii) Given that *B* lies on the line $y = x$, find the coordinates of *B*. **2**

(iii) Find the area of the trapezium *ABCD*. **2**

(iv) Find the coordinates of *C*. **2**

Question 12 continues on page 10

(c) The diagram shows the graphs of $y = x^2 + 2x - 5$ and $y = -2x$.
These two graphs intersect at point A and point B.



(i) Find the x -coordinates of the points of intersection A and B. **2**

(ii) Calculate the area of the shaded region. **3**

End of Question 12

Question 13 (15 marks) Use a **separate** writing booklet. **Marks**

(a) For the parabola $8x = 16y - y^2$.

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **1**

(iii) Sketch the curve showing all relevant features. **1**

(b) A man buys a new motorcycle. After t months its value $\$V$ is given by $V = 10\,000e^{-pt}$, where p is a constant.

(i) Find the value of the motorcycle when the man bought it. **1**

(ii) The value of the motorcycle after 12 months is expected to be \$4000. Calculate the expected value of the motorcycle after 18 months, correct to the nearest dollar. **3**

(iii) Calculate the age of the motorcycle, to the nearest month, when its expected value will be less than \$1000. **2**

(c) A particle moves in a straight line, so that, t seconds after leaving a fixed point O, its velocity, $v\text{ms}^{-1}$, is given by $v = \dfrac{12}{(t+1)^2} - 3$.

Find:

(i) an expression for the acceleration of the particle in terms of t. **2**

(ii) the distance travelled by the particle before it comes to instantaneous rest. **3**

End of Question 13

Question 14 (15 marks) Use a **separate** writing booklet. **Marks**

(a) (i) Use one application (two function values) of the trapezoidal rule to find an approximation to **2**

$$\int_0^2 \sqrt{16-x^2}\, dx.$$

(ii) Explain whether this approximation is greater than or less than the exact value. **1**

(b) Consider the function defined by $f(x) = x^3 + 3x^2 - 9x + 5$.

(i) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **3**

(ii) Find the coordinates of any point of inflexion. **2**

(iii) Sketch the graph of $f(x) = x^3 + 3x^2 - 9x + 5$ by showing the above information. **2**

(c) In its first year of production, 6000 mobile phones were sold by a company. Each year after that, sales were 15% more than the previous year's sales.

(i) Find the sales in the 10^{th} year of production. Express your answer to the nearest ten. **1**

(ii) Find the total sales in the first 10 years of production. Express your answer to the nearest ten. **2**

(iii) When will the total profit reach \$1 million if the company made \$10 profit on each sale? (Answer to the nearest whole year). **2**

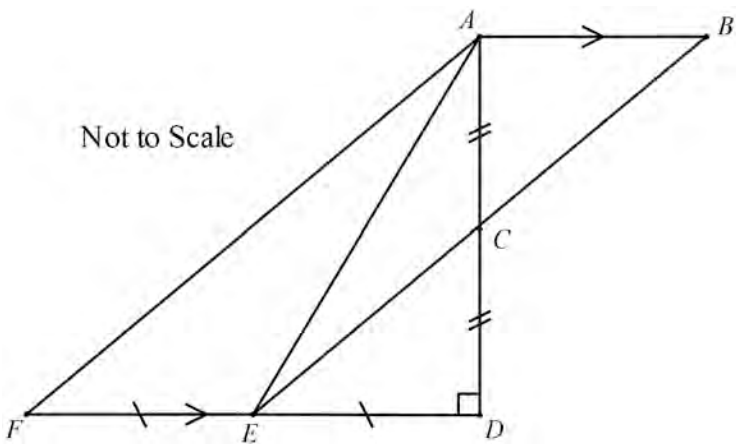
End of Question 14

Question 15 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Solve $\sin x = \cos x \quad for \;\; 0 \le x \le 2\pi.$ **2**

(b) Prove the identity $\dfrac{1}{1+\tan^2 A} = (1+\sin A)(1-\sin A).$ **2**

(c)



In the diagram $AB \| FD$, ADF is a right-angled triangle, C is the midpoint of AD and E is the midpoint of FD.

(i) Explain why $\angle CED = \angle ABC$. **1**

(ii) Show that $\Delta CDE \equiv \Delta CAB$. **2**

(iii) Show that $AF = 2BC$. **2**

Question 15 continues on page 14

(d) The roots of the quadratic equation $2x^2 + 4x + 5 = 0$ are α and β.

(i) Find the value of $\alpha + \beta$. **1**

(ii) Show that $\dfrac{\alpha^2 + \beta^2}{\alpha\beta} = -\dfrac{2}{5}$. **2**

(iii) The roots of $x^2 + mx + p = 0$ are $\dfrac{\alpha}{\beta} + 2$ and $\dfrac{\beta}{\alpha} + 2$. **3**

It is also given that $\dfrac{\alpha^2 + \beta^2}{\alpha\beta} = -\dfrac{2}{5}$.

Find the values of *m* and *p* where *m* and *p* are constants.

End of Question 15

Question 16 (15 marks) Use a **separate** writing booklet. **Marks**

(a) The depth D, in metres, of a liquid stored in a container at time t seconds is given by

$$D = \frac{t^2+1}{e^{2t}}, \qquad t \geq 0$$

(i) Find an expression for the rate at which the depth of the liquid changes. **2**

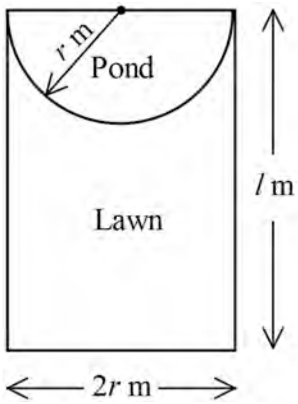
(ii) Hence, explain whether the depth of the liquid was increasing or decreasing at $t = 10$. **2**

(b) A curve has the equation $y = x^2 \log_e x$, where $x > 0$.

(i) Find an expression for $\frac{dy}{dx}$. **2**

(ii) Hence, find $\int x \log_e x \, dx$. **2**

(c) A garden is being designed to include a semi-circular pond in a rectangular shaped lawn. The radius of the pond is r metres and the length of the lawn is l metres, as shown in the diagram below.



(i) Given that the area of the lawn is 400 m^2, express l in terms of r. **2**

Show that $l = \frac{200}{r} + \frac{\pi}{4} r$.

(ii) Given that the perimeter of the lawn is P m, show that $P = \left(\frac{3\pi}{2} + 2\right) r + \frac{400}{r}$. **2**

(iii) Given that r and l can vary, find the value of r for which P is a minimum length. **3**

End of paper

HSC Trial Examination 2015

Mathematics - SOLUTIONS

SECTION I (10 marks)

1	B	6	D
2	C	7	D
3	B	8	D
4	B	9	A
5	A	10	B

SECTION II (90 marks)

Question 11: (15 marks)

a) $x^2 - 3 = 3x + 1$ 1 mark factorisation

/2 $x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ or -1. 1 mark

b) $3x - y = -7$ ① $\rightarrow$ $y = 3x + 7$

/2 $5x + 2y = 3$ ②

Sub. $y = 3x + 7$ into ②: $5x + 2(3x+7) = 3$

$11x + 14 = 3$

$11x = -11$

$x = -1$ 1 mark

Sub. $x = -1$ into $y = 3x + 7$

$= 3(-1) + 7$

$y = 4$ 1 mark.

Sol'n $\begin{cases} x = -1 \\ y = 4 \end{cases}$

c) $y = \dfrac{2x^3}{4x+2}$

/2 $= \dfrac{u}{v}$ where $u = 2x^3$; $v = 4x + 2$

$u' = 6x^2$ $v' = 4$

Q11

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{(4x+2).6x^2 - 2x^3.4}{(4x+2)^2}$$ 1 mark

$$= \frac{24x^3 + 12x^2 - 8x^3}{(4x+2)^2}$$ 1 mark

$$= \frac{4x^2(6x + 3 - 2x)}{(4x+2)^2}$$

$$y' = \frac{4x^2(4x+3)}{(4x+2)^2} \quad \text{or} \quad y' = \frac{4x^3+3x^2}{(2x+1)^2}$$

d) $\int_0^{\pi} \sin 2x \, dx = -\frac{1}{2}\left[\cos 2x\right]_0^{\pi}$ 1 mark

/2

$$= -\frac{1}{2}(\cos 2\pi - \cos 0)$$

$$= -\frac{1}{2}(1 - 1)$$

$$= 0$$ 1 mark

e) $\int_0^1 \frac{5}{\sqrt{e^x}} \, dx = \int_0^1 5e^{-\frac{x}{2}} \, dx$

/3

$$= 5 \times \left(\frac{+1}{-\frac{1}{2}}\right)\left[e^{-\frac{x}{2}}\right]_0^1$$

$$= -10\left(\frac{1}{\sqrt{e}} - \frac{1}{\sqrt{e^0}}\right)$$

$$= -10\left(\frac{1}{\sqrt{e}} - 1\right)$$

$$= -10\left(\frac{1}{\sqrt{e}} - 1\right)$$

3 marks all correct

2 marks correct integration plus correct substitution.

1 mark correct integration.

Q11 (3)

f) $A = \frac{1}{2} r^2 \theta$

Area $ABCD = \frac{1}{2}(x+7)^2 \cdot \frac{4}{3} - \frac{1}{2} \cdot 7^2 \cdot \frac{4}{3}$

$48 = \frac{2}{3}\left[x^2 + 14x + \cancel{49} - \cancel{49}\right]$

$72 = x^2 + 14x$

$0 = x^2 + 14x - 72$

$= (x+18)(x-4)$

$x = -18$ or 4

$\uparrow$

Disregard

$x > 0$

$\therefore$ Sol'n $x = 4$ cm

i $\quad OD = 7 + 4$

$/2 \quad = 11$ cm

2 marks all correct

1 mark – 1 error

ii $\quad P = 4 \times 2 + 7 \times \frac{4}{3} + 11 \times \frac{4}{3} \quad (l = r\theta)$

$/2$

$P = 32$ cm

2 marks all correct

1 mark – showed some understanding of perimeter being sum of 2 arcs and 2 radii

Question 12 (15 marks)

a) $2 - 1 + \frac{1}{2} \ldots$ $\qquad r = -\frac{1}{2}$, $a = 2$

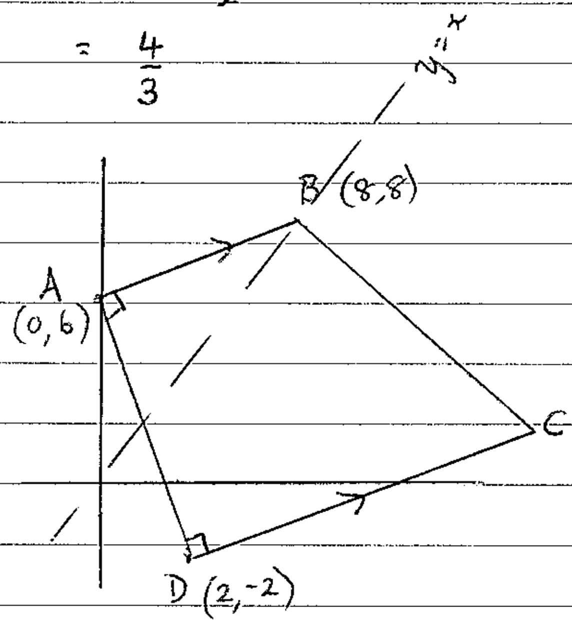
$\frac{1}{2}$ $\qquad (-1 < r < 1)$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{2}{1+\frac{1}{2}} \qquad \text{1 mark}$$

$$= \frac{4}{3} \qquad \text{1 mark}$$

b)



i $\quad$ AD: $\quad m_1 = \frac{-2-6}{2-0}$

$\frac{1}{2}$

$$= -4$$

AB: $\quad m_2 = \frac{1}{4}$ $\qquad$ 1 mark

Eq'n AB: $\quad \overset{x_1\ y_1}{(0, 6)}$ $\quad m = \frac{1}{4}$ $\qquad y - 6 = \frac{1}{4}(x - 0)$ $\qquad$ 1 mark

$4y - 24 = x$

$\therefore \quad 0 = x - 4y + 24$

ii $\quad$ B lies on $y = x$: $\quad 0 = x - 4y + 24$ $\qquad$ 1 mark

$\frac{1}{2}$

$$= x - 4x + 24$$

$$= -3x + 24$$

$$x = 8 \qquad \text{1 mark}$$

Coordinates of B: $(8, 8)$

Q12 $A(0,6)$ $B(8,8)$ $D(2,-2)$ ⑤

iii Area of trapezium $= \frac{h}{2}(a+b)$

/2

$AD = \sqrt{(2-0)^2 + (-2-6)^2}$
$= \sqrt{68}$

$AB = \sqrt{(8-0)^2 + (8-6)^2}$
$= \sqrt{68}$

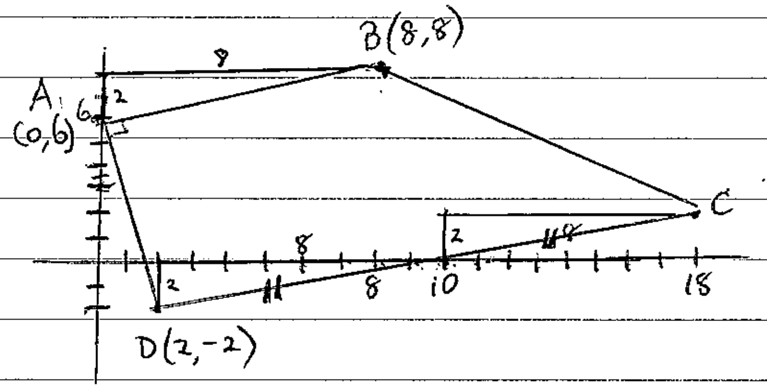
$DC = 2 \times AB = 2\sqrt{68}$
$= 4\sqrt{17}$

1 mark

Area $ABCD = \frac{\sqrt{68}}{2}\left(\sqrt{68} + 2\sqrt{68}\right)$
$= \frac{\cancel{2}\sqrt{17}}{\cancel{2}}\left(2\sqrt{17} + 4\sqrt{17}\right)$
$= 34 + 68$
$= 102$ units^2 1 mark

iv

/2



Coordinates of C : $(18, 2)$ 2 marks

c) i $y = x^2 + 2x - 5$ $y = -2x$

/2 $\therefore x^2 + 2x - 5 = -2x$ 1 mark
$x^2 + 4x - 5 = 0$
$(x+5)(x-1) = 0$
$x = -5$ and 1. 1 mark

x-coordinate of $A = -5$
x-coordinate of $B = 1$

1 mark 1 mark

ii $A_{-5}^{1} = \int_{-5}^{1} -2x - (x^2 + 2x - 5)\, dx$

/3

$= \int_{-5}^{1} -x^2 - 4x + 5 \; dx$

$= \left[\frac{-x^3}{3} - 2x^2 + 5x\right]_{-5}^{1}$

$= \left(-\frac{1}{3} - 2 + 5\right) - \left(\frac{125}{3} - 50 - 25\right)$

$= 36$ units^2. 1 mark.

(6)

Question 13 (15 marks)

a) i $8x = 16y - y^2$

$\frac{}{2}$

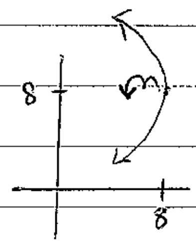
$y^2 - 16y + 8^2 = -8x + 8^2$

$(y-8)^2 = -8x + 64$

$(y-8)^2 = -8(x-8)$

$(y-k)^2 = -4a(x-h)$ 1 mark

$\therefore$ vertex $= (8, 8)$ 1 mark

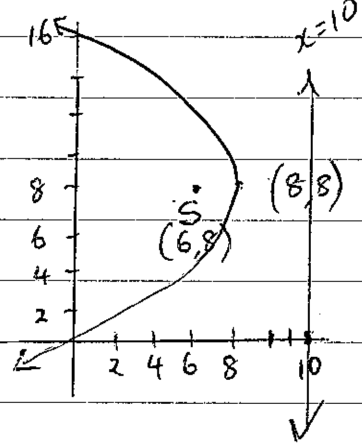


ii focal length $= 2$

1 focus $= (6, 8)$ 1 mark

iii

1



$x = 0 \rightarrow (y-8)^2 = 64$

$y - 8 = \pm 8$

$y = 16, 0$

1 mark

b) i $V = 10\,000e^{-pt}$

1 $V = V_0e^{-kt}$

$\therefore V_0 = 10\,000$

$\therefore$ Value of the motorcycle is \$10000 when the man bought it. 1 mark

ii Value after 12 months is \$4 000

3 $\rightarrow t = 1$ year ; $V = 4000$: $4000 = 10\,000e^{-p}$ 1 mark

$\frac{2}{5} = e^{-p}$

$\frac{5}{2} = e^{p}$

$\ln 2.5 = p$

$p \doteqdot 0.916\,291$ (6dp)

When $t = 1.5$ years : $V = 10000\,e^{-(0.916291 \times 1.5)}$ 1 mark

$= 2529.8221\ldots$

$\therefore$ Expected value after 18 months $=$ \$2 530 (nearest \$)

1 mark

Q13

b) iii $\frac{}{2}$ $V = 10\,000\, e^{-(\ln 2.5)t}$

$1000 > 10\,000\, e^{-(\ln 2.5)t}$

$\frac{1}{10} > e^{-(\ln 2.5)t}$

$10 < e^{(\ln 2.5)t}$ 1 mark

$\ln 10 < (\ln 2.5)t$

$\frac{\ln 10}{\ln 2.5} < t$

ie $t > 2.51294\ldots$

$\therefore$ Age of motorcycle when value is less than \$1000
= 2yrs 7months (0.6 of 12mths = 7.2 mths)

1 mark

c) i $\frac{}{2}$ $v = \frac{12}{(t+1)^2} - 3$

$a = \frac{dv}{dt}$

$V = 12(t+1)^{-2} - 3$

$\frac{dv}{dt} = -24(t+1)^{-3} \cdot 1$

$\therefore a = \frac{-24}{(t+1)^3}\ ms^{-2}$ 1 mark numerator
1 mark denominator

ii $\frac{}{3}$ $v = 12(t+1)^{-2} - 3$

$x = \frac{12(t+1)^{-1}}{(-1)(1)} - 3t + c$

$x = \frac{-12}{t+1} - 3t + c$

When $t=0, x=0$: $0 = -\frac{12}{1} - 0 + c$

$c = 12$

$\therefore x = \frac{-12}{t+1} - 3t + 12$

(continued over page)

Q13

c) ii cont'

At rest when $v = 0$:

$$0 = \frac{12}{(t+1)^2} - 3$$

$$0 = 12 - 3(t+1)^2$$

$$-12 = -3(t+1)^2$$

$$4 = (t+1)^2$$

$$t+1 = \pm 2$$

$$t = 1\text{ s or } -3\text{ s}$$ 1 mark

Disregard -ve time ($t \geqslant 0$)

When $t = 0$, $x = 0$ m

When $t = 1$, $x = \frac{-12}{(1)+1} - 3(1) + 12$

$= 3$ m

$\therefore$ Distance travelled is 3 metres. 1 mark

Question 14 (15 marks)

a) i

x	0	1	2
$f(x)$	4		$2\sqrt{3}$

$f(x) = \sqrt{16-x^2}$

/2

$\int_a^b f(x)\,dx \doteqdot \frac{b-a}{2}(f(a)+f(b))$

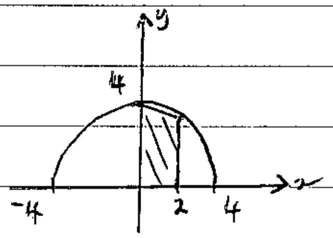
$\int_0^2 \sqrt{16-x^2}\,dx \doteqdot \frac{2-0}{2}\left(\sqrt{16}+\sqrt{12}\right)$ 1 mark

$\doteqdot 4 + 2\sqrt{3}$

$\doteqdot 7.46$ (2 d p) 1 mark

ii

/1



This approximation is less than the exact value as the trapezium formed by joining $f(0)$ to $f(2)$ lies below the curve. 1 mark

b) i $f(x) = x^3 + 3x^2 - 9x + 5$

/3 $f'(x) = 3x^2 + 6x - 9$

$f''(x) = 6x + 6$

Stat pts when $f'(x) = 0$

i.e. $3x^2 + 6x - 9 = 0$ 1 mark

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or 1

$(-3, 32)$

x	-4	-3	-2
$f'(x)$	15	0	-9
	/	—	\

is a MAX or $f''(-3) = -12$

$\therefore (-3, 32)$ is a Max

1 mark

$(1, 0)$

x	0	1	2
$f'(x)$	-9	0	15
	\	—	/

is a MIN or $f''(1) = 12$

$\therefore (1, 0)$ is a MIN

1 mark

$\therefore$ Stat pts at $(-3, 32)$ MAX and $(1, 0)$ MIN

Q14

(10)

b) ii inflexion pt when $f''(x)=0$

$/2$

i.e. $6x+6=0$ 1 mark

$x=-1$

$(-1,16)$

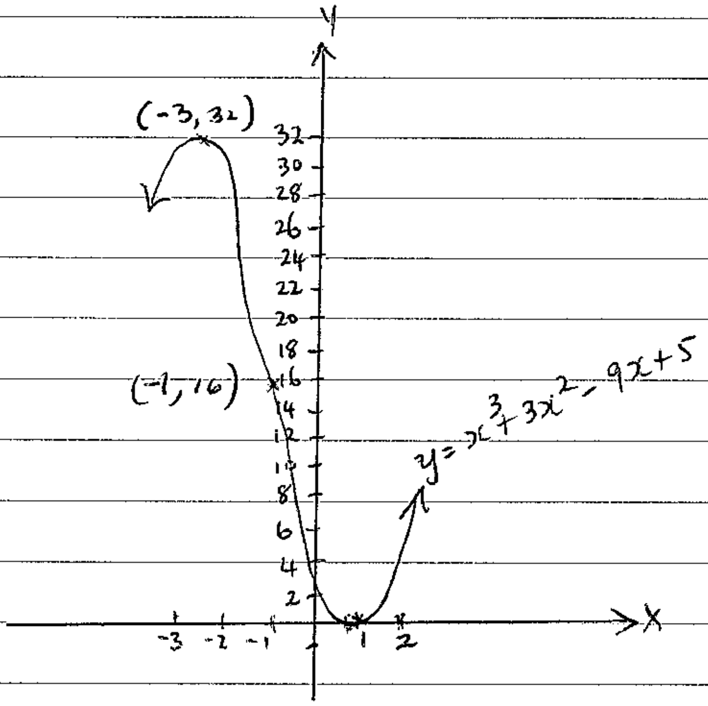
x	-2	-1	0
$f''(x)$	-6	0	6
	$\cap$	$-$	$\cup$

$\rightarrow$ change in concavity

$\therefore$ Pt of inflexion at $(-1,16)$ 1 mark

iii

$/2$



2 marks

c) i $6000,\ 6900,\ 7935,\ 9125.25, \ldots$

$/1$

$a=6000$

$r=1.15$

$T_n=ar^{n-1}$

$T_{10}=6000\times 1.15^9$

$=21107.25775$

$\doteqdot 21110$ (nearest ten) 1 mark

Sol'n: 21110 mobile phones sold in the 10^{th} year.

ii $S_n=\dfrac{a(r^n-1)}{r-1}$, $r>1$ $a=6000$

$/2$

$r=1.15$

$S_{10}=\dfrac{6000(1.15^{10}-1)}{0.15}$ 1 mark

$=121822.3094$ 1 mark.

Sol'n: Total sales in first 10 yrs = 121820 (nearest ten)

Q 14

(11)

c) iii $\$1\,000\,000 \div \$10 = 100\,000$ sales

½ $S_n = \dfrac{a(r^n - 1)}{r-1}$

$$100\,000 = \frac{6000\,(1.15^n - 1)}{0.15}$$ 1 mark

$$2.5 = 1.15^n - 1$$

$$3.5 = 1.15^n$$

$$\log 3.5 = n \log 1.15$$

$$n = \frac{\log 3.5}{\log 1.15}$$

$$= 8.963\,548\,64\ldots$$

$\therefore$ In the 9^{th} year sales will reach 100 000

$\therefore$ profits will reach \$1 000 000. 1 mark

Question 15 (15 marks)

a) $\sin x = \cos x \qquad 0 \leq x \leq 2\pi$

½ $\dfrac{\sin x}{\cos x} = 1$

$\tan x = 1$

S | A
T | C

$x = 45^\circ$ or $\dfrac{\pi}{4}$

$\tan x$ is positive, x is in 1st & 3rd quadrants

$\therefore x = \dfrac{\pi}{4}, \ \pi + \dfrac{\pi}{4}$

$\therefore x = \dfrac{\pi}{4}, \ \dfrac{5\pi}{4}$ 1 mark for each solution

b) Prove $\dfrac{1}{1+\tan^2 A} = (1+\sin A)(1-\sin A)$

½

LHS $= \dfrac{1}{1+\tan^2 A}$ $[\tan^2 A + 1 = \sec^2 A]$

$= \dfrac{1}{1+\sec^2 A - 1}$

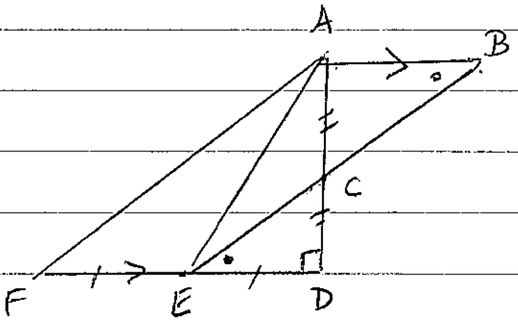
$= \dfrac{1}{\sec^2 A}$

$= \cos^2 A$ $[\sin^2 A + \cos^2 A = 1]$ 1 mark

$= 1 - \sin^2 A$

$= (1+\sin A)(1-\sin A)$ 1 mark

$=$ RHS



c) i $\angle CED = \angle ABC$

1/1 Alternate angles equal; $AB \parallel FD$

ii Prove $\Delta CDE \equiv \Delta CAB$

½

In Δ's CDE and CAB: (A) $\angle CED = \angle ABC$ (Alternate angles equal; AB||F

(A) $\angle ACB = \angle DCE$ (Vertically opposite angles)

(1) – 2 correct reasons

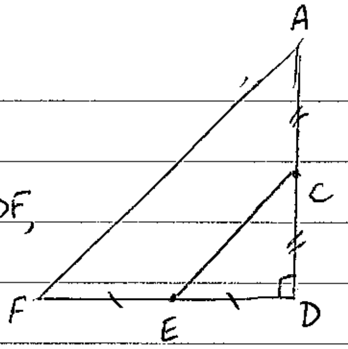
(S) $AC = DC$ (given C is midpt of AD)

(2) – All correct

$\therefore \Delta CDE \equiv \Delta CAB$ by AAS

Q15

c) iii Show $AF = 2BC$



$\frac{1}{2}$ In Δ's ADF and CDE:

$\frac{ED}{FD} = \frac{CD}{AD} = \frac{1}{2}$ (given E is midpt DF, C is midpt AD)

$\angle D$ is common

$\therefore \Delta ADF ||| \Delta CDE$ (2 pairs of corresponding sides in same ratio and included angle is equal) 1 mark

$\therefore AF = 2 \times CE$

$\therefore AF = 2 \times BC$ (from ii - corresponding sides CE and BC in congruent triangles CDE and CAB) 2 marks

d) $2x^2 + 4x + 5 = 0$

i $\alpha + \beta = -\frac{b}{a}$ $\quad a = 2,\ b = 4,\ c = 5$

$\frac{1}{1}$

$= -\frac{4}{2}$

$= -2$ 1 mark

ii Show $\frac{\alpha^2 + \beta^2}{\alpha\beta} = -\frac{2}{5}$

$\frac{1}{2}$

LHS $= \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$\alpha + \beta = -2$

$\alpha\beta = \frac{c}{a} = \frac{5}{2}$

$= \frac{\alpha^2 + 2\alpha\beta + \beta^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ 1 mark

$= \frac{(-2)^2 - 2\left(\frac{5}{2}\right)}{\frac{5}{2}}$ 1 mark

$= -1 \div \frac{5}{2}$

$= -\frac{2}{5}$

$=$ RHS

Q15

d) iii $x^2+ax+b=0$ Roots: $\frac{\alpha}{\beta}+2$, $\frac{\beta}{\alpha}+2$

$x^2+(\alpha+\beta)x+(\alpha\beta)=0$

$x^2+mx+p=0$ $\frac{\alpha}{\beta}+2$ $\frac{\beta}{\alpha}+2$

$\frac{\alpha^2+\beta^2}{\alpha\beta}=-\frac{2}{5}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+4=-m$

$\left(\frac{\alpha}{\beta}+2\right)\left(\frac{\beta}{\alpha}+2\right)=\frac{21}{5}$

$\left(\frac{\alpha}{\beta}+2\right)\left(\frac{\beta}{\alpha}+2\right)=p$

$\frac{2\alpha}{\beta}+\frac{2\beta}{\alpha}=-\frac{4}{5}$

$1+\frac{2\alpha}{\beta}+\frac{2\beta}{\alpha}+4=p$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=-\frac{2}{5}$

$\frac{2\alpha^2+2\beta^2}{\alpha\beta}=p-5$

$\therefore\ -\frac{2}{5}+4=-m$

$2\times-\frac{2}{5}=p-5$

$m=-\frac{18}{5}$

$p=5-\frac{4}{5}$

$=\frac{21}{5}$

1 mark

1 mark.

Question 16 (15 marks)

a) i $D = \frac{t^2+1}{e^{2t}}$ $t \geq 0$

½

$= \frac{u}{v}$ where $u = t^2+1$, $v = e^{2t}$

$u' = 2t$, $v' = 2e^{2t}$

$D' = \frac{vu' - uv'}{v^2}$

$= \frac{e^{2t}.2t - (t^2+1).2e^{2t}}{(e^{2t})^2}$ 1 mark

$= \frac{\cancel{e^{2t}}(2t - 2t^2 - 2)}{e^{\cancel{4t}\,2t}}$

$= \frac{-2(t^2 \cancel{t} + 1)}{e^{2t}}$ 1 mark

ii $t = 10$: $D' = \frac{-2(10^2+10+1)}{e^{20}}$

½

$= -\frac{182}{e^{20}}$ 1 mark

$\doteqdot -3.75 \times 10^{-7}$

< 0

$\therefore$ Depth of liquid decreasing at $t = 10$

1 mark

b) i $y = x^2 \log_e x$ $x > 0$

½

$= u.v$ where $u = x^2$, $v = \log_e x$

$u' = 2x$ $v' = \frac{1}{x}$

$y' = uv' + vu'$

$= x^2.\frac{1}{x} + \log_e x.2x$ 1 mark

$= x + 2x.\log_e x$ 1 mark

$= x(1 + 2\log_e x)$

Q16

b) ii $\int x \log_e x \, dx$

$/2$

$$\int (x + 2x \ln x)\, dx = x^2 \ln x$$

$$\int x \, dx + 2\int x \ln x \, dx = x^2 \ln x$$ 1 mark

$$2\int x \ln x \, dx = x^2 \ln x - \int x \, dx$$

$$\therefore \int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{2} + c$$

1 mark

c) i $A = 2rl - \dfrac{\pi r^2}{2}$

$/2$

$$400 = 2rl - \frac{\pi r^2}{2}$$ 1 mark

$$800 = 4rl - \pi r^2$$

$$4rl = 800 + \pi r^2$$

$$l = \frac{800}{4r} + \frac{\pi r^2}{4r}$$ 1 mark

$$l = \frac{200}{r} + \frac{\pi}{4} r$$

ii $P = \dfrac{2\pi r}{2} + 2l + 2r$

$/2$

$$= \pi r + 2r + 2\left(\frac{200}{r} + \frac{\pi}{4} r\right)$$ 1 mark

$$= \pi r + 2r + \frac{400}{r} + \frac{\pi}{2} r$$

$$= r\left(\pi + 2 + \frac{\pi}{2}\right) + \frac{400}{r}$$

$$\therefore P = \left(\frac{3\pi}{2} + 2\right) r + \frac{400}{r}$$ 1 mark

iii $\dfrac{dP}{dr} = \dfrac{3\pi}{2} + 2 + (-2).\,400\, r^{-2}$

$/3$

$$= \frac{3\pi}{2} + 2 - \frac{800}{r^2}$$

c) iii cont.

For stat pt, $\frac{dP}{dr} = 0$

$$\therefore \quad \frac{3\pi}{2} + 2 = \frac{400}{r^2} \qquad 1$$

$$\frac{3\pi + 4}{2} = \frac{400}{r^2}$$

$$r^2 = \frac{800}{3\pi + 4}$$

$$r = \pm\sqrt{\frac{800}{3\pi + 4}}$$

$r > 0$ since r is a length

$$\therefore \quad r = \sqrt{\frac{800}{3\pi + 4}} \doteqdot 7.72 \text{ m } (2dp)$$

1

$$\frac{d^2P}{dr^2} = -2\left(-400r^{-3}\right)$$

$$= \frac{800}{r^3}$$

When $r = 7.72$, $\frac{d^2P}{dr^2} = \frac{800}{7.72^3}$

$$> 0 \qquad 1$$

$\therefore P$ is a maximum when $r \doteqdot 7.72$m